12-8 Exponential and Binomial Distribution

Main Ideas

- Use exponential distributions to find exponential probabilities.
- Use binomial distributions to find binomial probabilities.

New Vocabulary

exponential distribution
exponential probability
binomial distribution
binomial probability

GET READY for the Lesson

The average length of time that a student at East High School spends talking on the phone per day is 1 hour. What is the probability that a randomly chosen student talks on the phone for more than 2 hours?

Exponential Distributions You can use exponential distributions to predict the probabilities of events based on time. They are most commonly used to measure *reliability*, which is the amount of time that a product lasts. Exponential distributions apply to situations where the time spent on an event, or the amount of time that an event lasts, is important.

Exponential distributions are represented by the following functions.

KEY CONCEPT *Exponential Distribution Functions*

The formula $f(x) = e^{-mx}$ gives the probability $f(x)$ that something lasts longer or costs more than the given value x, where m is the multiplicative inverse of the mean amount of time.

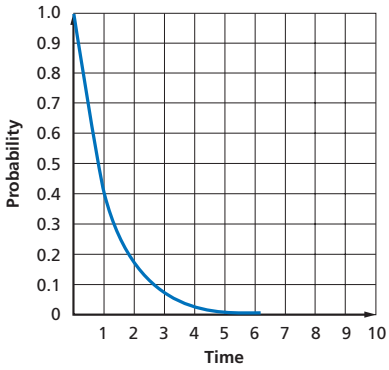
The formula $f(x) = 1 - e^{-mx}$ gives the probability $f(x)$ that something does not last as long or costs less than the given value x, where m is the multiplicative inverse of the mean amount of time.

Study Tip

Look Back

To review inverses, see Lesson 1–2.

Exponential distributions are represented by a curve similar to the one shown. The x-axis usually represents length of time, or money. The y-axis represents probability, so the range will be from 0 to 1.



EXAMPLE 1 **Exponential Distribution**

Refer to the application above. What is the probability that a randomly chosen student talks on the phone for more than 2 hours?

First, find the m, the inverse of the mean. Because the mean is 1, the multiplicative inverse is 1.

$f(x) = e^{-mx}$ Exponential Distribution Function

$f(2) = e^{-1(2)}$ Replace x with 2 and m with 1.

$= e^{-2}$ Simplify.

≈ 0.135 or 13.5% Use a calculator.

There is a 13.5% chance that a randomly selected East High School student talks on the phone for more than 2 hours a day. This appears to be a reasonable solution because few students spend either a short amount of time or a long amount of time on the phone.

CHECK Your Progress

1. If computers last an average of 3 years, what is the probability that a randomly selected computer will last more than 4 years?

EXAMPLE 2 Exponential Distribution

If athletic shoes last an average of 1.5 years, what is the probability that a randomly selected pair of athletic shoes will last less than 6 months?

The question asks for the probability that a pair of shoes lasts *less* than 6 months, so we will use the second exponential distribution function. The mean is 1.5 or $\frac{3}{2}$, so the multiplicative inverse m is $\frac{2}{3}$.

$f(x) = 1 - e^{-mx}$ Exponential Distribution Function

$f\left(\frac{1}{2}\right) = 1 - e^{-\frac{2}{3}\left(\frac{1}{2}\right)}$ Replace x with $\frac{1}{2}$ (6 mo $= \frac{1}{2}$ yr) and m with $\frac{2}{3}$.

$= 1 - e^{-\frac{1}{3}}$ Simplify.

≈ 0.2835 or 28.35% Use a calculator.

There is a 28.35% chance that a randomly selected pair of athletic shoes will last less than 6 months.

CHECK Your Progress

2. If the average lifespan of a dog is 12 years, what is the probability that a randomly selected dog will live less than 2 years?

Binomial Distributions In a binomial distribution, all of the trials are independent and have only two possible outcomes, success or failure. The probability of success is the same in every trial. The outcome of one trial does not affect the probabilities of any future trials. The random variable is the number of successes in a given number of trials.

KEY CONCEPT *Binomial Distribution Functions*

The probability of x successes in n independent trials is

$$P(x) = C(n, x)\, p^x\, q^{n-x},$$

where p is the probability of success of an individual trial and q is the probability of failure on that same individual trial ($p + q = 1$).

The expectation for a binomial distribution is

$$E(X) = np,$$

where n is the total number of trials and p is the probability of success.

EXAMPLE 3 Binomial Probability

A chocolate company makes boxes of assorted chocolates, 40% of which are dark chocolate on average. The production line mixes the chocolates randomly and packages 10 per box.

a. What is the probability that at least 3 chocolates in a given box are dark chocolates?

A success is a dark chocolate, so $p = 0.4$ and $q = 1 - 0.4$ or 0.6. You could add the probabilities of having exactly 3, 4, 5, 6, 7, 8, 9, or 10 dark chocolates, but it is easier to calculate the probability of the box having exactly 0, 1, or 2 chocolates and then subtracting that sum from 1.

$P(\geq 3 \text{ dark chocolates})$

$= 1 - P(< 3)$

$= 1 - P(0) - P(1) - P(2)$ Mutually exclusive events subtracted from 1

$= 1 - C(10, 0)(0.4)^0(0.6)^{10} - C(10, 1)(0.4)^1(0.6)^9 - C(10, 2)(0.4)^2(0.6)^8$

$= 0.8327$ or 83.27% Simplify.

The probability of at least three chocolates being dark chocolates is 0.8327 or 83.27%.

b. What is the expected number of dark chocolates in a box?

$E(X) = np$ Expectation for a binomial distribution

$= 10(0.4)$ $n = 10$ and $p = 0.4$

$= 4$ Multiply.

The expected number of dark chocolates in a box is 4.

CHECK Your Progress

3. If 20% of the chocolates are white chocolates, what is the probability that at least one chocolate in a given box is a white chocolate?

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CHECK Your Understanding

Examples 1, 2 (pp. 729–730)

For Exercises 1 and 2, use the following information.
The average amount of time high school students spend on homework is 2 hours per day.

1. What is the probability that a randomly selected student spends more than 3 hours per day on homework?

2. What is the probability that a randomly selected student spends less than 1 hour per day on homework?

Examples 3 (p. 731)

For Exercises 3 and 4, use the following information.
Mary's cat is having kittens. The probability of a kitten being male is 0.5.

3. If Mary's cat has 4 kittens, what is the probability that at least 3 will be male?

4. What is the expected number of males in a litter of 6?

Exercises

HOMEWORK HELP

For Exercises	See Examples
5-6, 9–14, 18–20	1–2
7–8, 15–17	3

For Exercises 5 and 6, use the following information.

The average life span of a certain type of car tire is 4 years.

5. What is the probability that a randomly selected set of 4 tires will last more than 9 years?
6. What is the probability that a randomly selected set of tires will last fewer than 2.5 years?

GARDENING **For Exercises 7 and 8, use the following information.**

Dan is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Dan knows that the probability of having a blue flower is 75%.

7. What is the probability that at least 20 of the flowers will be blue?
8. What is the expected number of white irises in Dan's garden?

Real-World Link

There are hundreds of species and cultivations of iris in all colors of the rainbow. Iris vary from tiny woodland ground covers, to 4-feet-tall flowers that flourish in the sun, to species that thrive in swampy soil. There is an iris that will do well in virtually every garden.

Source: hgic.clemson.edu

For Exercises 9–14, use the following information.

An exponential distribution has a mean of 0.5. Find each probability.

9. $x > 1.5$
10. $x > 3$
11. $x > \frac{1}{4}$
12. $x < 1$
13. $x < \frac{1}{3}$
14. $x < 2.5$

For Exercises 15–17, use the following information.

A binomial distribution has a 60% rate of success. There are 18 trials.

15. What is the probability that there will be at least 12 successes?
16. What is the probability that there will be 12 failures?
17. What is the expected number of successes?

RELIABILITY **For Exercises 18–20, use the following information.**

A light bulb has an average life of 8 months.

18. What is the probability that a randomly chosen bulb will last more than 13 months?
19. What is the probability that a randomly chosen bulb will last less than 6 months?
20. There is an 80% chance that a randomly chosen light bulb will last more than how long?

JURY DUTY **For Exercises 21–23, use the following information.**

A jury of twelve people is being selected for trial. The probability that a juror will be male is 0.5. The probability that a juror will vote to convict is 0.75.

21. What is the probability that more than 3 jurors will be men?
22. What is the probability that fewer than 6 jurors will vote to convict?
23. What is the expected number of votes for conviction?

EXTRA PRACTICE
See pages 919, 937.

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H.O.T. Problems

24. **OPEN ENDED** Sketch the graph of an exponential distribution function. Describe a situation in which you would expect data to be distributed in this way.

25. REASONING An exponential distribution function has a mean of 2. A fellow student says that the probability that $x > 2$ is 0.5. Determine whether this is *sometimes*, *always*, or *never* true. Explain your reasoning.

26. CHALLENGE The average amount of money spent per day by students in Mrs. Ross's class for lunch is \$2. In this class, 90% of students spend less than what amount per day?

27. *Writing in Math* Your school has received a grant, and the administration is considering adding a new science wing to the building. You have been asked to poll a sample of your classmates to find out if they support using the funding for the science wing project. Describe how you could use binomial distribution to predict the number of people in the school who would support the science wing project.

STANDARDIZED TEST PRACTICE

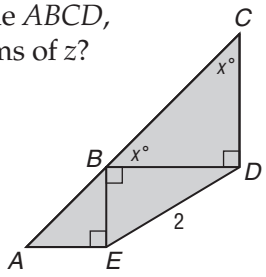
28. ACT/SAT In rectangle $ABCD$, what is $x + y$ in terms of z?

A $90 + z$

B $190 - z$

C $180 + z$

D $270 - z$



29. REVIEW Your gym teacher is randomly distributing 15 red dodge balls and 10 yellow dodge balls. What is the probability that the first ball that she hands out will be yellow and the second will be red?

F $\frac{1}{24}$

G $\frac{1}{4}$

H $\frac{2}{5}$

J $\frac{23}{25}$

Spiral Review

A set of 260 data values is normally distributed with a mean of 50 and a standard deviation of 5.5. (Lesson 12-7)

30. What percent of the data lies between 39 and 61?

31. What is the probability that a data value selected at random is greater than 39?

A die is rolled, Find each probability. (Lesson 12-5)

32. P(even)

33. P (1 or 6)

34. P(prime number)

Simplify each expression. (Lesson 6-2)

35. $(x - 7)(x + 9)$

36. $(4b^2 + 7)^2$

37. $(3q - 6) - (q + 13) + (-2q + 11)$

GET READY for the Next Lesson

PREREQUISITE SKILL Find the indicated term of each expression. (Lesson 11-7)

38. third term of $(a + b)^7$

39. fourth term of $(c + d)^8$

40. fifth term of $(x + y)^9$